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Bartlett, William Holmes
Chambers

Letter on life insurance to
Fred S. Winston

New York

1871

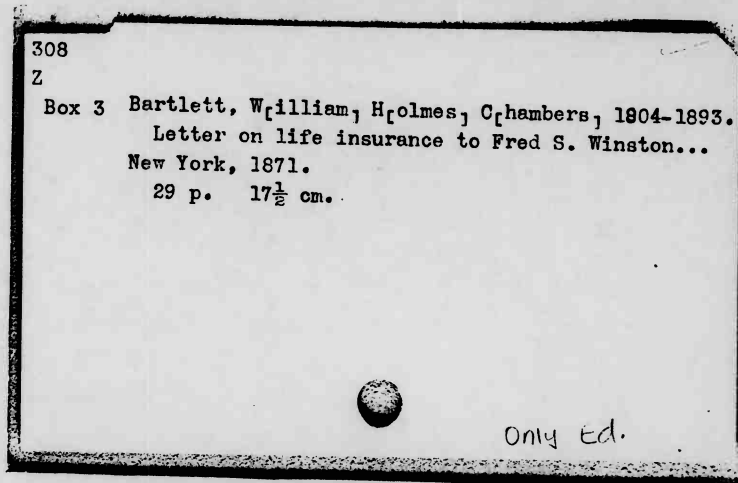
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For Wm. Morris Esqr

With kindest regards of Prof Bartlett

LETTER

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ON

LIFE ASSURANCE

TO

FRED. S. WINSTON, ESQ.,

PRESIDENT MUTUAL LIFE INSURANCE COMPANY OF NEW YORK.

BY

PROF. WM. H. C. BARTLETT.

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LETTER
ON
LIFE ASSURANCE.

WEST POINT, January 2, 1871.

TO FRED. S. WINSTON,

President Mutual Life Ins. Co. of New York.

Dear Sir: The recent failures of some well-known Insurance Companies, the variety of methods practiced by those still in existence, and other circumstances not important to designate, have caused a good deal of discussion in and out of the public prints; and the character of the discussion has seemed to me better suited to engender and stimulate to rapid growth a distrust of the principles of Life Assurance, than to vindicate their just claims to general confidence. Public opinion is not always founded in sufficient knowledge to be just, but is too often the result of honest ignorance misled by misrepresentation. Nothing seems too valuable or sacred, nowadays, for the schemes of daring empirics or the whims of thoughtless mediocrity. It would be a crime to deceive the large class of persons for whose benefit the institution of Life Assurance was specially devised, or to do anything which might have the effect to beckon them away from its advantages. It is the duty of all to do what they may to defend this noble charity, for such it is; and I have, therefore, thought it proper to address to you, as the head of the largest and most prosperous company in the country, perhaps in the world, this letter, in which I hope to explain the few and simple principles of Life Assurance in a way to bring them within the easy comprehension of all who need or desire this sort of information, and to demonstrate by implication the utter unsoundness of the assertions, I will not say arguments, too frequently arrayed against them.

The object of Life Assurance, as you well know, is to protect

from want those who lose their means of support by death. Vast numbers of widows and orphans are now drawing their only means of subsistence from its provident care; and still greater numbers are reposing in confidence upon its promises of future aid in time of need. It is one of the most beneficent gifts of civilization to the working classes, especially to salaried men, and is emphatically the poor man's friend. It shields his family from the chances of want in case of his sudden death, and only requires, as an antecedent condition, that he give it the *temporary* custody of a very small share of his current yearly earnings. It is to him a savings bank, and to them much more—an open-handed friend in time of bereavement.

PLAN.

The leading idea of Life Assurance may be thus briefly stated, viz.: Each of a number of persons pays into a common fund, either at once or by instalments, a certain sum called a *premium*, determined in amount by the condition that at death, or before, according to the agreement, his or her heirs shall receive from this fund, and its interest earnings, another specified sum, called a *reversion*. In case of early death, the sum paid to the heirs greatly exceeds that paid by the deceased, but this disparity contracts more and more as life continues, till, finally, the common fund becomes exempt from the chances of loss by accretions to the individual payments. It must not be supposed, however, that the heirs of *every* member will receive more than he pays; on the contrary, the members agree in effect, among themselves, that those whose good fortune it may be to have more than an average longevity, shall give of the excess of their payment and its gains to the support of the heirs of those who have less. Though based upon self-interest, the scheme is, as Professor de Morgan justly remarks, the most enlightened and beneficent form which the projects of self-interest ever took. To show how this idea may be put into practical execution with entire certainty, I shall be obliged to employ some elementary mathematics, but my letter shall be none the less easy of comprehension to the general reader on that account, since all its results will be translated into plain English.

SINGLE NET PREMIUM.

Take the following notation, viz.:

l_x = the number of persons, of age x , that unite to form a Mutual Life Company.

d_x = the number of those that die within any given portion of time, say a year.

i = rate of interest, or what one dollar will earn by being loaned out during the same unit of time.

R_x = the amount which each survivor, at the end of this unit of time, has to his credit at the beginning of the second unit of time.

l_{x+1} = the number living at the beginning of the second unit of time, or second year.

Π = the present value of a single and only payment of each member to secure a reversion of one dollar at death, called a single *net premium*.

Then the amount with which the company begins its business will be l_x times Π , or

$$l_x \cdot \Pi;$$

and this put out at interest will, at the end of the year, amount to

$$l_x \cdot \Pi \cdot (1+i);$$

from this is to be subtracted d_x dollars, for each death requires the payment of one dollar, and the sum with which the company begins its second year will be

$$l_x \cdot \Pi \cdot (1+i) - d_x;$$

and this must be equal to the amount of each survivor's credit at the beginning of the second year, multiplied by the number of survivors; whence the equation:

$$l_x \cdot \Pi \cdot (1+i) - d_x = l_{x+1} \cdot R_{x+1}. \quad (1.)$$

The amount $l_{x+1} \cdot R_{x+1}$, or its equal $l_x \cdot \Pi \cdot (1+i) - d_x$, being put out at interest, will amount, at the end of the second year, to

$$[l_x \cdot \Pi \cdot (1+i) - d_x] (1+i),$$

and this, diminished by the payment of the death claims, amount-

ing to d_{x+1} , this latter denoting the number that die during the second year, will give, as before, the amount of funds with which the company begins the third year. But this is equal to the amount standing to the credit of each survivor at the beginning of the *third* year, denoted by R_{x+2} , multiplied by the number of survivors, denoted by l_{x+2} , and we shall have

$$[l_x \cdot \Pi \cdot (1+i) - l_x] (1+i) - l_{x+1} = l_{x+2} \cdot R_{x+2},$$

or,

$$l_x \cdot \Pi \cdot (1+i)^2 - l_x (1+i) - l_{x+1} = l_{x+2} \cdot R_{x+2}. \quad (2.)$$

Again, putting out at interest the sum $l_{x+2} \cdot R_{x+2}$, or its equal, in the first member, and subtracting from its amount at the end of the third year the death claims of that year, denoted by d_{x+3} , we have, by suitably changing the subscript notation,

$$l_x \cdot \Pi \cdot (1+i)^3 - l_x (1+i)^2 - l_{x+1} (1+i) - l_{x+2} = l_{x+3} \cdot R_{x+3}; \quad (3.)$$

in which the law is developed and manifest; so that if $x+n$ denote the oldest age to which any member of the company may reach, we may write, generally,

$$l_x \cdot \Pi \cdot (1+i)^{n+1} - l_x (1+i)^n - l_{x+1} (1+i)^{n-1} \dots - l_{x+n} = l_{x+n+1} \cdot R_{x+n+1}.$$

But $x+n$ being the greatest age, there can be none of $x+n+1$, and hence $l_{x+n+1} = 0$; which will reduce the preceding equation to

$$l_x \cdot \Pi \cdot (1+i)^{n+1} - l_x (1+i)^n - l_{x+1} (1+i)^{n-1} \dots - l_{x+n} = 0. \quad (4.)$$

Now, just as certainly as the mortality conforms to the rates d_x , d_{x+1} , d_{x+2} , &c., and the interest to the rate i , will the last dollar be paid to the heirs of the last survivor, aged $x+n$, and the fund and the company become extinct together—after paying the claims of the heirs of the other members as they mature in the lapse of time; the only condition on the part of the assured being the payment by each, at the outset, of the single premium Π . Of the certainty of these rates, more presently.

To find the value of Π , solve equation (4), and we have

$$\Pi = \frac{1}{1+i} \cdot \frac{d_x}{l_x} + \frac{1}{(1+i)^2} \cdot \frac{d_{x+1}}{l_x} + \frac{1}{(1+i)^3} \cdot \frac{d_{x+2}}{l_x} + \dots + \frac{1}{(1+i)^{n+1}} \cdot \frac{d_{x+n}}{l_x};$$

or making

$$\Pi = v \cdot \frac{d_x}{l_x} + v^2 \cdot \frac{d_{x+1}}{l_x} + v^3 \cdot \frac{d_{x+2}}{l_x} + \dots + v^{n+1} \cdot \frac{d_{x+n}}{l_x}. \quad (5.)$$

Or, multiplying both numerator and denominator of all the terms in the second member by v^n , which will not alter the value,

$$\Pi = \frac{v^{x+1} \cdot d_x + v^{x+2} \cdot d_{x+1} + v^{x+3} \cdot d_{x+2} + \dots + v^{x+n+1} \cdot d_{x+n}}{v^n l_x}, \quad (6.)$$

which is the formula employed in the construction of what are called *commutation tables*, for the easy and expeditious determination of single net premiums for assurances of one dollar at death. For any other reversion this premium has only to be multiplied by the number of dollars in the reversion.

ANNUAL NET PREMIUM.

So much for a single premium. But far the more common mode of making payments, by the assured, is by what are called net annual premiums during life, or for a shorter period—the principle being the same in both. Take the case of annual payments during life, and denote the annual premium by π . Then, from what has already been explained in detail, it will be easily seen that the mathematics will stand thus, viz.:

For the first year, as before,

$$l_x \cdot \pi \cdot (1+i) - l_x = l_{x+1} \cdot R_{x+1}.$$

There being l_{x+1} , living at the beginning of the second year, and as each pays another premium π , the company will begin the second year with the amount,

$$l_{x+1} \cdot R_{x+1} + l_{x+1} \cdot \pi,$$

or with its equal,

$$l_x \cdot \pi \cdot (1+i) + l_{x+1} \cdot \pi - l_x;$$

and this at interest during the second year will amount, at its close, to

$$[l_x \cdot \pi \cdot (1+i) + l_{x+1} \cdot \pi - d_x] \cdot (1+i);$$

and this, diminished by the death claims, amounting to d_{x+1} dollars, during the year, will give

$$\pi \cdot [l_x (1+i)^2 + l_{x+1} (1+i)] - d_x (1+i) - d_{x+1} = l_{x+2} \cdot R_{x+2};$$

In like manner, the amount of $l_{x+2} \cdot R_{x+2} + l_{x+3} \cdot \pi$, or its equal, obtained by replacing $l_{x+2} \cdot R_{x+2}$ by its value in the first member of the last equation, diminished by the death claims of the third year, amounting to d_{x+2} dollars, will give

$$\pi [l_x (1+i)^3 + l_{x+1} (1+i)^2 + l_{x+2} (1+i)] - d_x (1+i)^2 - d_{x+1} (1+i) - d_{x+2} = l_{x+3} \cdot R_{x+3};$$

and here, the law being manifest, we may write, generally,

$$\pi [l_x (1+i)^{n+1} + l_{x+1} (1+i)^n + l_{x+2} (1+i)^{n-1} \dots + l_{x+n-1} (1+i)] - [d_x (1+i)^n + d_{x+1} (1+i)^{n-1} \dots + d_{x+n}] = l_{x+n+1} \cdot R_{x+n+1};$$

but, as before, $l_{x+n+1} = 0$, and solving with respect to π ,

$$\pi = \frac{d_x (1+i)^n + d_{x+1} (1+i)^{n-1} + d_{x+2} (1+i)^{n-2} \dots + d_{x+n-1} (1+i) + d_{x+n}}{l_x (1+i)^{n+1} + l_{x+1} (1+i)^n + l_{x+2} (1+i)^{n-1} \dots + l_{x+n-1} (1+i)};$$

dividing the terms in both numerator and denominator by $l_x \cdot (1+i)^{n+1}$, and writing

$$v \text{ for } \frac{1}{1+i},$$

we find

$$\pi = \frac{v \cdot \frac{d_x}{l_x} + v^2 \cdot \frac{d_{x+1}}{l_x} + v^3 \cdot \frac{d_{x+2}}{l_x} \dots + v^n \cdot \frac{d_{x+n-1}}{l_x} + v^{n+1} \cdot \frac{d_{x+n}}{l_x}}{1 + v \cdot \frac{l_{x+1}}{l_x} + v^2 \cdot \frac{l_{x+2}}{l_x} + v^3 \cdot \frac{l_{x+3}}{l_x} \dots + v^n \cdot \frac{l_{x+n}}{l_x}}, \quad (7.)$$

which gives the ordinary rule for finding the net annual premium. And here, it may be remarked, as in the case of a single premium, it only requires the anticipated rates of mortality and of interest to be realized, to secure to the heirs of the assured the prompt payment of the company's obligations as they mature.

RATES OF MORTALITY AND OF INTEREST.

Now, an examination of equations (5) and (7) shows that the premiums increase in value as the rate of mortality increases and that of interest decreases. Safety to the company is, therefore, secured by assuming, in the computation of premiums, a rate of mortality higher, and of interest lower, than those ever likely to be realized.

Careful records of births and deaths, extended through a long series of years, in different countries, have revealed the laws of mortality among people composed of all classes and possessed of the ordinary means and comforts of life. These laws are defined in what are called mortuary tables, and the ratios of d_x to l_x and l_{x+1} to l_x , for all ages, tabulated for easy reference and use. These laws being employed in the computations for *assured lives, which are always selected*, and of which the rates of mortality are always less than those of the community at large, will satisfy one of the conditions of safety.

The rate of interest is also suggested by experience; and being taken below that received on the actual loans and other business operations of the community where the funds of the company are to be employed, will secure the other condition of safety.

Thus, assuming as a basis of computation a higher rate of mortality than the company will realize among its members, and a lower rate of interest than it will get upon its loans, the premiums will be higher than necessary, and the objects sought by the formula secured beyond all reasonable doubt.

PRESENT AND FUTURE OBLIGATIONS.

Let me apply formula (6) to an hypothetical case for the purpose of illustration. Suppose a company of 1000 persons aged 20; and take the rates of your own office, in which $i=0.04$; $20+n=95$; or $n=75$, and we find

$$\Pi_{20} = \$0.24776;$$

and, therefore,

$$l_{20} \cdot \Pi_{20} = 1000 \times 0.24776 = \$247.76;$$

so that, with a capital of two hundred and forty-seven dollars and

seventy-six cents, at the outset, the company engages to pay one thousand dollars in the course of seventy-five years, which it does with its original capital and its interest earnings.

Again, if the members of the same company engage to pay by instalments a net annual premium during life, which is the more common case, we shall find.

$$\pi_a = \$0.01267,$$

and

$$l_{25}, \pi_a = 1000 \times 0.01267 = \$12.67.$$

That is, one thousand persons beginning business at the age of 20, with a capital of twelve dollars and sixty-seven cents, and each paying into the common fund but a trifle over a cent and a quarter annually, will give to the heirs of each member at death, one dollar. The last payment, which will be made in the seventy-fifth year of the company's existence, will just exhaust the funds, and at the very moment the company becomes extinct from the limitation of human existence.

The actual value of the company's obligations, at any time, is measured by the amount of death claims then matured, and can never be as great as the company's assets, except at the death of the last surviving member, when they become exactly equal.

You often hear persons, who talk without thought, employ language which has the effect to condense, as it were, into a single day the successive obligations of a company that can, by the very terms which bring them into existence, only mature through a long series of years, and no more rapidly than the assets to meet them, and thus produce the most erroneous and damaging impressions. Such persons will, without any regard to the objects and offices of verbal tense, say, for instance, that my hypothetical company has outstanding obligations amounting to a thousand dollars, while its assets are but a little over twelve; and assert, with equal emphasis, as a consequence, that it is helplessly bankrupt—a conclusion, to use no harsher terms, utterly illogical, and true in no sense whatever. The fact is, no one but those having access to a company's books can know anything of its *actual* obligations. Your own company has now assured to the amount of two hundred and forty-two millions of dollars, and its present assets are under forty-five millions, and to infer that

you are, therefore, bankrupt, would be about as wise as to conclude that the Croton river is unreliable because the amount of water *now* running between its banks is insufficient for the *future* as well as present supply of your city.

Again, and I write from experience, these persons will argue that local epidemic diseases suggest the advent of others so wide-spread and devastating as to render all computations founded upon the observed laws of human vitality unreliable and worthless. And so do rain showers suggest another Noachian deluge to drown all the living, and earthquakes another general eruption of the internal fires to burn them up! The one suggestion is about as significant as the other. Such people have no belief that a law of nature may be detected by observation and experience, however well directed and long continued.

ANNUITIES.

It is unnecessary to my purpose to give the demonstrations for the various rules by which are computed the premiums appropriate to the great variety of assurance policies. They are equally dependent upon the same elementary formula with which I began. But as many companies take annuity risks, I will add the demonstration of the rule for finding the present value or price of an *annuity*.

An *annuity* is a specific sum of money paid to an individual at stated periods of time, say at the end of every year, in consideration of another sum paid down by the recipient, called an *annuitant*, or other person in the annuitant's behalf.

Suppose a company of persons, all of the same age x , and consisting of l_x members, to pay into a common fund a sum sufficient to give each an annuity of one dollar, to be paid annually at the end of the year during their natural life. What price shall each pay, the rate of interest being i , and the mortality rates those adopted by any company? Denote this price by the usual notation a_x , the number living at the age x by l_x ; $x+1$ by l_{x+1} ; $x+2$ by l_{x+2} , etc.

The funds of the company, at the end of the first year, will amount, according to what has already been explained, to

$$l_x \cdot a_x (1+i) - l_{x+1} = l_{x+1} \cdot R_{x+1};$$

and at the end of the second year to

$$[l_x \cdot a_x \cdot (1+i) - l_{x+1}] (1+i) - l_{x+2} = l_{x+2} \cdot R_{x+2},$$

or

$$l_x \cdot a_x (1+i)^2 - l_{x+1} (1+i) - l_{x+2} = l_{x+2} \cdot R_{x+2}$$

at the end of the third year,

$$l_x \cdot a_x \cdot (1+i)^3 - l_{x+1} (1+i)^2 - l_{x+2} (1+i) - l_{x+3} = l_{x+3} \cdot R_{x+3};$$

or generally, since the law of the series is manifest,

$$l_x \cdot a_x \cdot (1+i)^n - l_{x+1} (1+i)^{n-1} - l_{x+2} (1+i)^{n-2} \dots - l_{x+n-1} (1+i) - l_{x+n} = l_{x+n} \cdot R_{x+n}.$$

But as none will be alive after the $(x+n)^{\text{th}}$ year of age, the payment to those who reach that age must be the last, and will exhaust the funds, so that $R_{x+n} = 0$; and we find

$$l_x \cdot a_x \cdot (1+i)^n - l_{x+1} (1+i)^{n-1} - l_{x+2} (1+i)^{n-2} \dots - l_{x+n-1} (1+i) - l_{x+n} = 0;$$

and solving with respect to a_x , and making

$$r = \frac{1}{1+i},$$

we get

$$a_x = r \cdot \frac{l_{x+1}}{l_x} + r^2 \cdot \frac{l_{x+2}}{l_x} + r^3 \cdot \frac{l_{x+3}}{l_x} \dots + r^n \cdot \frac{l_{x+n}}{l_x}; \quad (8.)$$

or multiplying both numerator and denominator of each term in the second member by r^n , which will not alter its value, we have

$$a_x = \frac{r^{n+1} \cdot l_{x+1} + r^{n+2} \cdot l_{x+2} \dots + r^{n+n} \cdot l_{x+n}}{r^n \cdot l_x}; \quad (9.)$$

which is the price sought, and under the form employed in computing the commutation tables for annuities on single lives.

THE TEMPORARY COMPANY MADE PERPETUAL.

Now analyze formulas (5) and (8): taking the first of these, the first term of the second member is

$$r \cdot \frac{d_x}{l_x};$$

in which r denotes the present value of one dollar at the end of a year; that is, it is the sum which, put out at interest at the rate i , will grow to one dollar in one year. But the payment of this dollar is contingent upon the death of the assured, and the dollar itself is called the *contingent gain* to the heirs of the assured, or contingent loss to the company. The numerator of the fraction into which it is multiplied, to wit, d_x , is the number out of l_x that die at age x , and when divided by l_x , gives the ratio of the number that favor death to the whole number that favor and oppose it, and is, therefore, the probability that a particular person named in advance will die at the age x . The product of the contingent gain by the probability that the event will happen to cause the loss or gain, is the present value of what is called the *expectation*.

In the same way may the second term of the second member be shown to be the present value of the expectation of loss or gain at the end of the second year, and so on; so that we have this rule for finding the present value of a single net premium of assurance, viz.: Find the present value of the expectations of loss to the company for all the years from date of policy to end of human life, as given by the mortuary tables, and take their sum.

Again, $\frac{l_{x+1}}{l_x}$ is the measure of the probability that any designated individual will live through his x^{th} year to receive his annuity. And a similar analysis of formula (8) tells us that the rule by which to find the present value of an annuity for life, is to find the present values of the expectations of the annuitant from date to the end of human life, and take the sum. And a reference to formula (7) will show this rule for finding the annual net premium, viz.: Divide the single net premium by the value of an annuity of one dollar increased by unity.

These rules are perfectly independent of the actual number of members in a company, and are strictly applicable to single cases; so that it is not necessary, as I assumed at the outset, that the members shall be of the same age, but they may be of all ages and admitted at all times, a sufficient number only are required to make sure of the laws of human mortality when the

company begins its business. My hypothetical and temporary company becomes, therefore, converted into a perpetual one. The rates of mortality and of interest are prepared in advance, and have only to be applied in the manner just explained, to ascertain the sum or sums to be paid on the admission of any member.

In this way ideas of probability and of chance came to be associated with life assurance, and gave it, to those unacquainted with its real character, an aspect of uncertainty which it does not, in reality, possess. The uncertainty attaches to individuals, not to large collections of individuals. It would be very unwise, for instance, to assure one individual, and no more, upon the conditions of formulas (5) and (7), and yet perfectly safe to assure a hundred thousand, each on those conditions.

RESERVES.

The reserve of any member is his working insurance capital. At the date of joining the company, it is his net premium. It augments as time lapses by the accumulation of interest, and diminishes by the loss of its share of the death claims. It is continually varying in value, but, on the whole, expands more than it contracts, till finally, at the close of the longest life of the table, should it pertain to such life, it amounts to the sum assured. The sum total of all the reserves is a true measure of what the company's assets ought to be. If the actual assets fall below this sum, the company is insolvent and should be wound up by re-assuring in some responsible company to the extent of the reserves available for this purpose. If the assets be in excess of the sum total of the reserves, it is a sign of healthy condition, and that the members have overpaid, and the excess should be distributed in the manner to be described presently. By a study of the mathematical processes I have given, it will be apparent that a mutual company performs both the functions of a savings bank and an insurance institution. It receives, cares for, and adds interest to the premiums; and converts, from time to time, this interest, in part, into capital; while it also takes the vital risks of the assured, and, in case of death, pays to his heirs his individual reserve, and, in addition, the excess of the sum assured

above this reserve, from the accumulating funds of the company, of which the balance goes to make up the reserves of the surviving members. An individual reserve, which is the true money value of the policy, and in which the holder and the company have a joint interest, has been very properly called the amount of a member's self-insurance. It is, so to speak, the tangible substance of the policy, and should be at all times within certain reach of the company.

All persons intrusted with the interest of others should be held to a stringent accountability, and be made to render, at short and stated intervals, an account of their stewardship; and this is emphatically true of the officers of an insurance company. The Legislature of this State, acting upon this principle, has very wisely required, as one of the conditions of your own charter, for instance, that you strike a balance on a given day of every year, which shall exhibit the actual state of every member's account at that time, and show the precise amount of your reserves, actual assets and future liabilities, that the members of the company, and those who may desire to become members, may form a just estimate of your actual condition. Then, every member's reserve should be accurately computed up to this day, the sum of the whole taken, and this sum compared with the actual and well-ascertained assets. To compute these reserves, resume the elementary formula (1), viz.:

$$l_x \cdot \Pi_x \cdot (1+i) - d_x = l_{x+1} \cdot R_{x+1};$$

in which R_{x+1} represents the reserve of each member that begins his $(x+1)^{\text{th}}$ year; and had all the members entered the company at the same age, and on the legal day of settlement, it would only be necessary to find the value of R_{x+1} , from this equation in terms of the known values which compose it. But we have just shown that members join at almost all ages, and at all times, and assuming that the rate of mortality is continuous, and denoting by t the fractional part of the year from date of policy to the day of reckoning, we may write the above equation thus, viz.:

$$l_x \cdot \Pi_x (1+i)^t - t \cdot d_x = (l_x - t \cdot d_x) \cdot R_{x+t};$$

whence

$$R_{x+t} = \frac{l_x \cdot \Pi_x \cdot (1+i)^t - t \cdot d_x}{l_x - t \cdot d_x},$$

or

$$R_{x+t} = \frac{(1+i)^t}{d_x} \cdot \left\{ \Pi - \frac{1}{(1+i)^t} \cdot \frac{d_x}{l_x} \right\}, \quad (10.)$$

or

$$R_{x+t} = \frac{1+i \cdot i}{d_x} \cdot \left\{ \Pi - \frac{1}{1+i \cdot i} \cdot \frac{d_x}{l_x} \right\}. \quad (11.)$$

But for the use we shall make of the reserve formula for the fractional part of the year, the form (10) will perhaps be the more convenient.

Making $t=1$, we have

$$R_{x+1} = \frac{1+i}{d_x} \cdot \left\{ \Pi - \frac{1}{1+i} \cdot \frac{d_x}{l_x} \right\}; \quad (12.)$$

and generally for a single net premium,

$$R_{x+n} = \frac{1+i}{d_{x+n-1}} \cdot \left\{ R_{x+n-1} - \frac{1}{1+i} \cdot \frac{d_{x+n-1}}{l_{x+n-1}} \right\}; \quad (13.)$$

and for an annual premium,

$$R_{x+n} = \frac{1+i}{d_{x+n-1}} \cdot \left\{ R_{x+n-1} + \pi - \frac{1}{1+i} \cdot \frac{d_{x+n-1}}{l_{x+n-1}} \right\}. \quad (14.)$$

For any fractional part of the year at any distance, as $n-1$, from that of entrance, equation (10) may be written

$$R_{x+n-1+t} = \frac{(1+i)^t}{d_{x+n-1}} \cdot \left\{ R_{x+n-1} - \frac{1}{(1+i)^t} \cdot \frac{d_{x+n-1}}{l_{x+n-1}} \right\} \quad (15.)$$

for a single net premium; and for an annual premium.

$$R_{x+n-1+t} = \frac{(1+i)^t}{d_{x+n-1}} \cdot \left\{ R_{x+n-1} + \pi - \frac{1}{(1+i)^t} \cdot \frac{d_{x+n-1}}{l_{x+n-1}} \right\} \quad (16.)$$

From the equations (13) or (14), according as the case pertains to a single or annual premium, compute the reserves from anniversary to anniversary of the policy, and enter them in a table arranged in the form of double entry, the arguments being the ages of the members at time of entrance, in a horizontal series of caption spaces, separated by vertical lines when the sheet is held upright, and the ages of the policies arranged in the left hand vertical column, each pair of consecutive whole years being separated by nine horizontal lines for the reception of interpolated values for tenths of years, determined by the use of equations (15) or (16), as the case may be. On the right of every vertical column of reserve values there should be a column for differences, which should also be entered.

With the aid of this table, the reserve on any policy may be ascertained for any day of the policy's existence, with the same ease and accuracy that the logarithm of a number may be taken from a table of logarithms. The sum total of all the reserves may be determined long before the close of the fiscal year, ready for comparison with the assets as soon as known, and the question of solvency answered. If, perchance, some of the included reserves may have disappeared by post-mortem settlement, these may easily be subtracted from the result. Such a table will last as long as the company adheres to any given or assumed set of rates, both of interest and mortality, and should be re-computed as soon as these are changed.

EXPENSES AND LOADING.

The only payments the company has thus far been required to make have been for death claims, and the explanations have been confined to that branch of a company's operations which pertains strictly to assurance. But there are many incidental expenses, such as office rent, salaries of officers, lawyers' and doctors' fees,

clerk hire, fuel, stationery, and the like, to be cared for. These are defrayed from a fund created by adding to each net premium a certain percentage of its own value, and requiring the assured to pay it at the time of paying his premium. This addition, called *loading*, amounts to from 20 to 40 per cent., according to circumstances. The funds of the company, above the amount required for immediate death claims and current expenses, are loaned out at the market rate of interest, upon undoubted securities, of which the intrinsic value is generally double the sum loaned.

I do not here speak of the expenses of agencies, because they have nothing to do with the real business of assurance. They will be referred to further on.

SURPLUS AND ITS DISTRIBUTION.

If the expenses of a company be just equal to the sum total of the loadings; if the rate of interest on its loans and losses from death be just equal to those assumed in the computations of net premiums, the assets, at the close of the fiscal year, will be just equal to the sum total of the reserves. But this, in well-managed companies, rarely happens, and almost always the gross loadings exceed the expenses, the rate of interest is higher than that anticipated, and the losses from mortality less; and hence the assets are generally greater than the reserves. The excess of the former over the latter is what is called *surplus*, and is the amount by which the members, collectively, have been overcharged; and is therefore returned, in equitable proportions, not in money, but in the form of credits for paid up policies, of which the reversionary amounts are determined by the principles already explained for single net premiums and loadings.

Thus, make

- X_{x+t} , = the share of a member, aged $x+t$;
 m = the percentage of X_{x+t} , which goes to the loading.
 Π_{x+t} , = net single premium to assure one dollar at age $x+t$;
 S_{x+t} , = reversionary value of paid up policy;

Then,

$$X_{x+t} - mX_{x+t} = (1-m) \cdot X_{x+t} = \text{net sum to pay for policy};$$

and as there must be as many dollars in the reversion as there are net dollar premiums in the net purchase money, we have

$$S_{x+t} = (1-m) \cdot X_{x+t} \cdot \frac{1}{\Pi_{x+t}} = (1-m) \frac{X_{x+t}}{\Pi_{x+t}}. \quad (17.)$$

This new policy, the property of an old member, will, at the close of every fiscal year of its existence, have its reserves, and, also, its credits or debits, according as the company is prospering or the reverse. In some years the expenses may be small, the interest on its loans high, and the mortality below that of the tables, in which case the surplus would be large and the reversionary credits correspondingly large. In other years just the reverse may happen, and, indeed, there might be a deficiency instead of a surplus, and this should be met by a reversionary debit to this fluctuating account, always leaving the original policy intact.

MODE OF DISTRIBUTION.

The only equitable method of distribution is that known as the *contribution plan*. As its name indicates, it aims to return to each member the amount of his actual contribution to the surplus. It has been a good deal discussed, much praised, and, in my opinion, not too much; and yet it is susceptible of very easy misapplication, and of being made the means of wrong as well as of right. The following demonstration will explain it:

Resume the elementary equation

$$l_x \cdot \pi \cdot (1+i) - d_x = l_{x+1} \cdot R_{x+1} = (l_x - d_x) \cdot R_{x+1}.$$

This embraces one unit of time, as a year. And for any fractional portion of a year, denoted by t ,

$$l_x \cdot \pi \cdot (1+t \cdot i) - t \cdot d_x = (l_x - t \cdot d_x) \cdot R_{x+t};$$

or which is the same thing,

$$\pi \cdot (1+t \cdot i) - t \cdot \frac{d_x}{l_x} (1 - R_{x+t}) - R_{x+t} = 0; \quad (18.)$$

that is, the amount of net premium at end of time t , diminished by its share of death claims and the reserve at same epoch, must leave nothing.

Now make

λ = loading;

e = individual yearly expense;

i = rate of interest realized;

d' = yearly number of deaths realized at age x ;

then substitute $\Pi + \lambda - t \cdot e$ for Π ; i' for i ; d'_x for d_x , and we get

$$(\pi + \lambda - te)(1+i't) - t \cdot \frac{d'_x}{l_x} (1 - R_{x+t}) - R_{x+t} = X_{x+t}, \quad (19.)$$

in which X_{x+t} is the contribution at the end of t . Subtracting equation (18) from this, and diminishing the difference by the reserve loading $(1-t)\lambda$, we find

$$t \left\{ \pi(i' - i) + \lambda - e + (\lambda - et)i' + (1 - \frac{d'_x}{d_x}) \cdot \frac{d_x}{l_x} (1 - R_{x+t}) \right\} = X_{x+t}, \quad (20.)$$

and using the sign Σ to denote the operation of summing,

$$\Sigma \lambda : \Sigma e :: \lambda : e,$$

or

$$e = \lambda \cdot \frac{\Sigma e}{\Sigma \lambda};$$

and denoting the actual loss from deaths by a , and that of the tables or imputed by ϵ , we may write

$$\frac{d'_x}{d_x} = \frac{a}{\epsilon};$$

and substituting in equation (20), we get

$$t \left\{ \pi(i' - i) + \lambda \cdot \left\{ 1 + i' - \frac{\Sigma e}{\Sigma \lambda} \cdot (1 + i't) \right\} + (1 - \frac{a}{\epsilon}) \cdot \frac{d_x}{l_x} \cdot (1 - R_{x+t}) \right\} = X_{x+t}, \quad (21.)$$

or, perhaps, sufficiently near

$$t \cdot \left\{ \pi(i' - i) + \lambda \cdot \left\{ 1 + i' - \frac{\Sigma e}{\Sigma \lambda} \cdot (1 + i') + (1 - \frac{a}{\epsilon}) \cdot \frac{d_x}{l_x} \cdot (1 - R_{x+t}) \right\} \right\} = X_{x+t}. \quad (21.)'$$

The sum of the actual expenses and that of the loadings will be known from the books of the company. The ratio

$$\frac{\Sigma e}{\Sigma \lambda}$$

will therefore be known, and the factor $\frac{\Sigma e}{\Sigma \lambda} \cdot \frac{d'_x}{d_x}$ becomes constant for every policy for same settlement. The ratio $\frac{a}{\epsilon}$ for which $\frac{a}{\epsilon}$ is

substituted, is the ratio of the actual to the expected deaths at age x , or the ratio of the actual to the expected loss from death, supposing each assured for one dollar. In strictness,

the ratio $\frac{d'_x}{d_x}$ might be employed, but it would work great hard-

ship, if not downright injustice, in many cases. Supposing, for instance, an epidemic disease should prevail particularly fatal to persons of a certain age, and not to those who are younger and older, the loss from death would fall heavily upon the survivors of that age. Indeed, in their case the actual loss from death might be greater than the expected, and make the factor

$$1 - \frac{d'_x}{d_x},$$

which will presently appear, negative; while if the actual number of deaths in the whole company should be less than the expected, this factor, for other members, would be positive, thus deranging the general rates of mortality assumed at the outset. On the principle of protection implied in the mutual system, the actual losses and benefits arising from temporary departures from general laws should be shared alike by the whole company, and it would be much more in accordance with the principles of

equity to make the ratio $\frac{a}{\epsilon}$ a general term, applicable to all pol-

icies. To do this, a should denote the actual loss from death of the whole company during the year, and ϵ the expected loss from same cause.

The actual loss of the whole company during the year is the

amount of death claims diminished by that of the reserves of the deceased members, and is therefore known from the books. Thus, denoting a single death claim, at age x , by D_x , and the reserve of same at end of year, on one dollar, by R_{x+1} , and making similar notation for other ages, we have

$$a = \Sigma D_x (1 - R_{x+1}). \quad (22.)$$

The expected loss is also easily found thus: denote by L_x the sum assured on any life of age x , and by R_{x+1} , the corresponding reserve, and make similar notation for other ages, then will

$$e = -\frac{d_x}{l_x} \Sigma L_x (1 - R_{x+1}) + \dots + \frac{d_{x+n-1}}{l_{x+n-1}} \Sigma L_{x+n-1} \cdot (1 - R_{x+n-2});$$

or more generally and concisely,

$$e = -\Sigma \frac{d_x}{l_x} \cdot \Sigma L_x (1 - R_{x+1}). \quad (23.)$$

That is, take the sum of the amounts assured for each age, diminish this sum by that of the reserves for that age, and multiply the remainder by the probability of death at that age, then take the sum of the products—the result will be the *expected* loss.

Equation (21) will give to a member his appropriate share of surplus at the close of the fiscal year of entrance, t being the fractional portion of the year from date of policy to that epoch. Making $t=1$, that equation becomes

$$\pi(i-i) + \lambda \cdot (1 - \frac{\Sigma e}{\Sigma \lambda}) \cdot (1+i') + (1 - \frac{a}{\epsilon}) \cdot \frac{d_x}{l_x} \cdot (1 - R_{x+1}) = X_{x+1}, \quad (24.)$$

which is the appropriate formula for individual share of surplus at the end of the first policy year, provided no settlement be made before. But this formula should never be used unless the company is at liberty to have just as many days of general settlement as there are policy anniversaries.

Now the account which was settled at the end of the fiscal year, or at the end of t , begins the next *entire* fiscal year with the re-

serve R_{x+t} , and with the loading $(1-t) \cdot \lambda$, and has added to its credit at the end of the policy year, and at the distance t in time, from the close of the next fiscal year, the sum $\pi + \lambda$, in case of an annual premium, or only λ , if a single premium. Supposing the former, for the present—the latter will be considered presently—the formula becomes

$$[R_{x+t} + (1-t) \cdot \lambda - e] \cdot (1+i') + (\pi + \lambda) \cdot (1+ti') - \frac{d_x}{l_x} \cdot (1 - R_{x+1+t}) - ?_{x+1+t} \\ = X_{x+1+t};$$

but employing the net premium and rates of interest and of mortality of the table,

$$R_{x+t}(1+i) + \pi(1+ti) - \frac{d_x}{l_x} \cdot (1 - R_{x+1+t}) - R_{x+1+t} = 0,$$

subtracting this, and diminishing the difference by the reserve loading $(1-t)\lambda$, we find

$$[R_{x+t} + t \cdot \pi] \cdot (i' - i) + \lambda \cdot (1 - \frac{\Sigma e}{\Sigma \lambda}) \cdot (1+i') + (1 - \frac{a}{\epsilon}) \cdot \frac{d_x}{l_x} \cdot (1 - R_{x+1+t}) = \\ X_{x+1+t}$$

which gives the surplus due to the assumed policy at the close of the first entire fiscal year of membership; that is, if the day of settlement be the 31st of December, it will give the surplus at the end of the first calendar year after that of entrance. And generally, for the surplus at the close of the n^{th} calendar year after entrance, we may write

$$[R_{x+n-1+t} + t \cdot \pi] \cdot (i' - i) + \lambda \cdot (1 - \frac{\Sigma e}{\Sigma \lambda}) \cdot (1+i') + (1 - \frac{a}{\epsilon}) \cdot \frac{d_{x+n-1}}{l_{x+n-1}} \cdot (1 - \\ R_{x+n+t}) = X_{x+n+t} \quad (25.)$$

in which the factors $(i' - i)$, $(1 - \frac{\Sigma e}{\Sigma \lambda})$ and $(1 - \frac{a}{\epsilon})$ are constant; that is, the same for all policies for the same day of reckoning.

When a member dies, and sufficient time has been allowed to collect the evidence of the circumstances of death, the amount

standing to his credit at the date of last settlement is usually paid to his heirs. But this does not exhaust his interest in the company. At the next day of general adjustment of accounts, his special account is closed, and the balance which may stand to his credit on that day is paid. Formulas for these post-mortem credits and payments may be easily deduced from equation (15).

ANNUAL LOADING FOR SINGLE PREMIUM.

It has been the practice in some quarters to load a single premium by the addition of a percentage, and to make this loading contribute to expenses for one year and no more. To do this according to the method required by equation (25) would be very unjust, and a gross violation of the principles of equity. Obviously, the proper way would be, in all cases of single premiums, to ascertain the value of the corresponding annual premium to assure the same sum, take the percentage on this, and then load the single premiums by a sum which would give an annuity for life equal to this percentage. This percentage would be the value of λ , to be used annually, in equation (25), during the policy's existence

If the assurance be purchased by a limited number of annual payments, the case would be somewhat different, though the principle would be the same. The following process will lead to the rule to suit all cases of whole life policies, whether the payment be by single, several, or annual premiums. Make

Λ =the loading on each of the limited number of payments;

λ =the equivalent loading on an annual premium for life.

y =greatest age of the tables;

x =age of the assured at entrance;

$y-x=n$;

m =number of annual payments.

Then

$(\Lambda-\lambda)(1+i)=\Lambda(1+i)-\lambda(1+i)=\text{accumulation at end of first year};$

$$\left\{ \Lambda(1+i) + \Lambda \frac{l_{x+1}}{l_x} - \lambda(1+i) - \lambda \frac{l_{x+1}}{l_x} \right\} \cdot (1+i), \text{ or}$$

$$\Lambda \left\{ (1+i)^2 + \frac{l_{x+1}}{l_x} \cdot (1+i) \right\} - \lambda \left\{ (1+i)^2 + \frac{l_{x+1}}{l_x} \cdot (1+i) \right\} = \text{accumulation at end of second year.}$$

and generally, stopping the accumulation at the end of the $(n-1)^{\text{th}}$ year, and omitting the terms involving Λ , after the m^{th} payment, and because the last annual loading, λ , exhausts the loadings Λ , and their accumulations, we have

$$\Lambda \left\{ (1+i)^{n-1} + (1+i)^{n-2} \cdot \frac{l_{x+1}}{l_x} + \dots + (1+i)^{n-1} \cdot \frac{l_{x+m-1}}{l_x} \right\} - \lambda \left\{ (1+i)^{n-1} + (1+i)^{n-2} \cdot \frac{l_{x+1}}{l_x} + \dots + (1+i)^{n-1} \cdot \frac{l_{x+m-1}}{l_x} \right\} = 0;$$

or

$$\Lambda \cdot \frac{(1+i)^n}{1+i} \cdot \left\{ 1 + \frac{1}{1+i} \cdot \frac{l_{x+1}}{l_x} + \dots + \frac{1}{(1+i)^{n-1}} \cdot \frac{l_{x+m-1}}{l_x} \right\} - \lambda \cdot \frac{(1+i)^n}{1+i} \cdot \left\{ 1 + \frac{1}{1+i} \cdot \frac{l_{x+1}}{l_x} + \frac{1}{(1+i)^{n-1}} \cdot \frac{l_{x+m-1}}{l_x} \right\} = 0,$$

omitting the common factor, transposing and multiplying the numerator and denominator of each term, except the first in each

bracket, by $v^x = \frac{1}{(1+i)^x}$, we find

$$\Lambda \cdot \left\{ 1 + v^{x+1} \cdot \frac{l_{x+1}}{l_x} + v^{x+2} \cdot \frac{l_{x+2}}{l_x} + \dots + v^{x+m-1} \cdot \frac{l_{x+m-1}}{l_x} \right\} = \lambda \cdot \left\{ 1 + v^{x+1} \cdot \frac{l_{x+1}}{l_x} + v^{x+2} \cdot \frac{l_{x+2}}{l_x} + \dots + v^{x+m-1} \cdot \frac{l_{x+m-1}}{l_x} \right\}.$$

But

$$v^{x+1} \cdot \frac{l_{x+1}}{l_x} + v^{x+2} \cdot \frac{l_{x+2}}{l_x} + \dots + v^{x+m-1} \cdot \frac{l_{x+m-1}}{l_x} = a_{\overline{m}|i} = \text{the price of a}$$

temporary annuity of one dollar for $m-1$ years; and

$n^{x+1} \cdot \frac{l_{x+1}}{l_x} + n^{x+2} \cdot \frac{l_{x+2}}{l_x} + \dots + n^{x+n-1} \cdot \frac{l_{x+n-1}}{l_x} = \frac{l_{x+n-1}}{l_x} =$ the price of a temporary annuity of one dollar for $n-1$ years. Whence

$$\Lambda[1 + i'_{x, n-1}] = \lambda[1 + i'_{x, n-1}]$$

or

$$\Lambda = \lambda \frac{1 + i'_{x, n-1}}{1 + i'_{x, n-1}} \quad (26.)$$

If $m=1$, which is the case of a single premium, then

$$i'_{x, m-1} = i'_0 = 0, \text{ and}$$

$$\Lambda = \lambda[1 + i'_{x, n-1}].$$

If $m=n$, which gives the case of an annual loading on an annual premium for life, then

$$1 + i'_{x, n-1} = 1 + i'_{x, n-1}, \text{ and}$$

$$\Lambda = \lambda.$$

As before remarked, the annual premium should first be found, and its appropriate loading determined. This will give λ , which, substituted in equation (26), will give the value of Λ to be charged upon a single or limited number of premiums, as the case may be.

The factor $\frac{l_{x+n}}{l_{x+n}}$, which measures the probability that any one

designated member, whose age is $x+n$, will die in the current year, is only true in proportion as the mortality tables are true. But these, at best, are but approximations to the truth—close, to be sure, but still, only approximations. And again, there is no absolute certainty that in all cases the assured knows his own age. Here, then, is a range of uncertainty quite broad enough for the assumption that any slight change in this factor, less than half a year, would as probably prove an approach to as a departure from the real truth. And as it is desirable to free the equation for surplus from all ambiguity in its application, and to diminish

the number of its terms to save labor, it is suggested that in all cases the anniversary of the date of natural birth, as designated by the assured, be made coincident with that of the anniversary of settlement nearest thereto. This would have the effect to increase the affix numbers to the subscript x by unity, at the close of every calendar year.

DISTRIBUTION INTEREST FACTOR.

One of the most important operations an actuary has to perform is, as we have just seen, to divide a given surplus among the policy holders of a company upon the principles of equity; that is, upon the contribution plan. The formulas for this have just been explained. But there is a very important feature about these formulas yet to be considered. An actuary may attempt the division, and carry his work to a conclusion, and yet find that he has not attained his object. He may not have divided all his surplus, or he may have divided more than his surplus. The operation has, from its very nature, been tedious to the computers and expensive to the company, and failure becomes a serious matter; because, in addition, it produces uneasiness and dissatisfaction among the policy holders. The question now is, how shall failure be prevented with certainty? The answer is, by finding a suitable value for the interest factor, i' , before the process of division is begun. To this end it is remarked, that if the surplus arose only from diminished mortality and expenses, and increased and uniform interest on loans, all the quantities in the first member of equation (25) would be known; and, therefore, the second member or individual share of surplus would be known. But there are other sources that pour their proceeds into the general surplus, such as forfeitures, rents, sales of investments, etc., etc., and as these additional contributions may be regarded as interest gained upon the company's working capital, a rate of interest should be found which will embrace *them* as well as the earnings on money actually loaned. This rate once found, and placed in equation (25) for i' , will secure the exact division of the surplus. The equation (25) considers the sum assured to be one dollar, and when multiplied by the sum assured, called L , and performing the op-

erations indicated, as regards the factor i , it is found, after transposing all the terms which are independent of i to the second member, that

$$L \cdot \left\{ R_{x+n-1, t+t} \pi + \lambda \left(1 - \frac{\Sigma e}{\Sigma \lambda}\right) \right\} i = L \cdot X_{x+n} + L \cdot \left\{ R_{x+n-1, t+t} \pi \right\} \cdot i \\ - \left(1 - \frac{\Sigma e}{\Sigma \lambda}\right) \cdot L \cdot \lambda - \left(1 - \frac{a}{e}\right) \cdot \frac{d_{x+n-1}}{l_{x+n-1}} \cdot L \cdot (1 - R_{x+n});$$

and conceiving all the equations for all the policies to be written out in this way and added together, we get, by employing the sign Σ to indicate, as before, the process of summation,

$$i \cdot \Sigma L \cdot \left\{ R_{x+n-1, t+t} \pi + \lambda \left(1 - \frac{\Sigma e}{\Sigma \lambda}\right) \right\} = \Sigma L \cdot X_{x+n} + i \Sigma L \cdot \left\{ R_{x+n-1, t+t} \pi \right\} \\ - \left(1 - \frac{\Sigma e}{\Sigma \lambda}\right) \cdot \Sigma \lambda L - \left(1 - \frac{a}{e}\right) \cdot \Sigma \frac{d_{x+n-1}}{l_{x+n-1}} \cdot L \cdot (1 - R_{x+n}).$$

But $\Sigma L \cdot X_{x+n} = X$ = sum to be divided. Substituting this, and solving with respect to i , there will result

$$X + i \cdot \Sigma L \cdot \left[R_{x+n-1, t+t} \pi - \left(1 - \frac{\Sigma e}{\Sigma \lambda}\right) \cdot \Sigma \lambda L - \left(1 - \frac{a}{e}\right) \cdot \Sigma \frac{d_{x+n-1}}{l_{x+n-1}} \cdot L \cdot (1 - R_{x+n}) \right] \\ i = \frac{\Sigma L \cdot \left\{ R_{x+n-1, t+t} \pi + \lambda \left(1 - \frac{\Sigma e}{\Sigma \lambda}\right) \right\}}{\quad} \quad (27.)$$

whence this rule to find the interest factor that will divide a given surplus, viz : Increase the sum to be divided by the tabular interest on the reserves at the beginning of the year and the net premiums ~~received~~ during the year; diminish the sum by the surplus loadings and charges for mortality losses; divide the difference by the sum of the reserves at the beginning of the year, increased by the surplus loadings and the sum of the fractional premiums that carry the policies from their anniversary dates to end of current fiscal year. The value of X is found from the equation

$$X = A - \Sigma R_x - \Sigma (1 - l) \cdot \left(\frac{\Sigma e}{\Sigma \lambda} \right) - \Sigma D, \quad (23.)$$

in which A denotes the gross assets; ΣR_x , the sum of the reserves to end of fiscal year; $\Sigma (1 - l) \cdot \left(\frac{\Sigma e}{\Sigma \lambda} \right)$ the sum of the unexpended gross premiums ~~of premiums~~ that carry the policies to the ends of their anniversary years; and ΣD the sum of unpaid death claims, or other claims that may not have been adjusted.

I have dwelt the longer on this mode of dividing surplus—known as the contribution plan—because of its intrinsic merits, and of the ease with which it may, without great caution, be applied to aggravate the evils it was intended, and is so admirably suited, to remedy.

I have now finished the task I had proposed to myself, to wit : that of running two of the more common and popular policies of assurance through a mutual company for the purposes of explanation and illustration. It was not in my intention to write a treatise on life assurance, but merely to demonstrate the reliability of its simple principles by actual examples; and I think I am justified in the conclusion, that no one can study and understand what I have written, and avoid the conclusion, that of all kinds of business, that of life assurance is, when properly managed, the safest and surest; and that the wisest and most prudent disposition a person of moderate income, and having a family to care for, can make of his little savings, is to invest them in a policy of assurance with some company known to conduct its affairs upon true assurance principles.

If a life assurance company fail, you may with certainty look for the cause in some gross actuarial mistakes, wasteful and unnecessary expenditures, or gross mismanagement of its funds—perhaps all of these combined. A company suffering under all of these ills has but one alternative, and that is, to wind up by reinsuring, as far as its assets will permit, in some other responsible company, and pass to the category of things that were. But this remark is just as applicable to all corporations as to insurance companies.

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